PROOF OF FORMULA 4.271.4

$$\int_0^1 \frac{\ln^{p-1} x \, dx}{1-x} = (-1)^{p-1}\Gamma(p)\zeta(p)$$

Expand the integrand in a geometric series to obtain

$$\int_0^1 \frac{\ln^{p-1} x \, dx}{1-x} = \sum_{j=0}^{\infty} \int_0^1 x^j \ln^{p-1} x \, dx.$$

The change of variable $x = e^{-u}$ produces

$$\int_0^1 \frac{\ln^{p-1} x \, dx}{1-x} = \sum_{j=0}^{\infty} (-1)^{p-1} \int_0^{\infty} e^{-(j+1)u} u^{p-1} \, du.$$

The result now follows from the change of variable $t = (j+1)u$.