PROOF OF FORMULA 3.194.4

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(1+bx)^{n+1}} = (-1)^n \frac{\pi}{b^\mu \, \sin(\pi\mu)} \binom{\mu-1}{n}$$

Let $t = bx$ to obtain

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(1+bx)^{n+1}} = \frac{1}{b^\mu} \int_0^\infty \frac{t^{\mu-1}\,dt}{(1+t)^{n+1}}.$$

The integral representation

$$B(a,b) = \int_0^\infty \frac{t^{a-1}\,dt}{(1+t)^{a+b}}$$

gives

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(1+bx)^{n+1}} = \frac{1}{b^\mu} B(\mu, n+1-\mu).$$

Now use $\Gamma(n+1-\mu) = \Gamma(1-\mu)(1-\mu)_n$ and

$$(1-\mu)_n = (-1)^n \frac{(\mu-1)!}{(\mu-n+1)!}$$

and $\Gamma(\mu)\Gamma(1-\mu) = \pi/\sin(\pi\mu)$ to simplify the answer.