

PROOF OF FORMULA 3.194.6

$$\int_0^\infty \frac{x^{\mu-1} dx}{(1+bx)^2} = \frac{(1-\mu)\pi}{b^\mu \sin(\pi\mu)}$$

Formula 3.194.3 states that

$$\int_0^\infty \frac{x^{\mu-1} dx}{(1+bx)^\nu} = b^{-\mu} B(\mu, \nu - \mu).$$

The special case $\nu = 2$ gives

$$\int_0^\infty \frac{x^{\mu-1} dx}{(1+bx)^2} = b^{-\mu} B(\mu, 2 - \mu).$$

The result comes from the reduction

$$B(\mu, 2 - \mu) = \frac{\Gamma(\mu)\Gamma(2 - \mu)}{\Gamma(2)} = \Gamma(\mu)(1 - \mu)\Gamma(1 - \mu) = \frac{(1 - \mu)\pi}{\sin \pi\mu}.$$