PROOF OF FORMULA 3.196.5

$$\int_{-\infty}^{1} \frac{dx}{(a-bx)(1-x)^{\nu}} = \frac{\pi}{b \sin \pi\nu}(a/b-1)^{-\nu}$$

Let $t = 1-x$ to obtain

$$\int_{-\infty}^{1} \frac{dx}{(a-bx)(1-x)^{\nu}} = \int_0^{\infty} \frac{dt}{[(a-b)+bt]\, t^{\nu}}.$$

The change of variables $s = \frac{b}{a-b}t$ gives

$$\int_0^{\infty} \frac{dt}{[(a-b)+bt]\, t^{\nu}} = \frac{1}{b}\left(\frac{b}{a-b}\right)^{\nu} \int_0^{\infty} \frac{ds}{s^{\nu}(1+s)}.$$

The identity

$$B(u,v) = \int_0^{\infty} \frac{t^{u-1}\, dt}{(1+t)^{u+v}},$$

shows that

$$\int_0^{\infty} \frac{ds}{s^{\nu}(1+s)} = B(1-\nu,\nu).$$

The result now follows from the expressins

$$B(u,v) = \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)} \text{ and } \Gamma(u)\Gamma(1-u) = \frac{\pi}{\sin \pi u}.$$