PROOF OF FORMULA 3.197.6

$$\int_1^\infty x^{\lambda-\nu}(x-1)^{\nu-\mu-1}(ax-1)^{-\lambda}\,dx = a^{-\lambda}B(\mu,\mu-\nu)\ {}_2F_1\left[\lambda,\mu;\nu;\tfrac{1}{a}\right]$$

The change of variables $t = 1/x$ gives

$$\int_1^\infty x^{\lambda-\nu}(x-1)^{\nu-\mu-1} = a^{-\lambda}\int_0^1 t^{\mu-1}(1-t)^{\nu-\mu-1}(1-t/a)^{-\lambda}\,dt.$$

The result now follows from the integral representation of the hypergeometric function

$${}_2F_1\left[\alpha,\beta;\gamma;z\right] = \frac{1}{B(\beta,\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-zx)^{-\alpha}\,dx.$$