PROOF OF FORMULA 3.219

$$\int_0^\infty \left(\frac{x^\nu}{(x+1)^{\nu+1}} - \frac{x^\mu}{(x+1)^{\mu+1}} \right) dx = \psi(\mu+1) - \psi(\nu+1)$$

Let $t = 1/x$ to obtain

$$\int_0^\infty \left(\frac{x^\nu}{(x+1)^{\nu+1}} - \frac{x^\mu}{(x+1)^{\mu+1}} \right) dx = \int_0^\infty \left(\frac{1}{(1+t)^{\nu+1}} - \frac{1}{(1+t)^{\mu+1}} \right) \frac{dt}{t}.$$

The result now follows from the representation $(8.361, 6)$:

$$\psi(z) = \int_0^\infty \left(\frac{1}{1+t} - \frac{1}{(1+t)^{z+1}} \right) \frac{dt}{t} - \gamma.$$