PROOF OF FORMULA 3.221.1

$$\int_a^\infty \frac{(x-a)^{p-1}}{x-b}\,dx = \frac{\pi(a-b)^{p-1}}{\sin \pi p}$$

Let $t = (x-a)/(b-a)$ to obtain

$$\int_a^\infty \frac{(x-a)^{p-1}}{x-b}\,dx = (a-b)^{p-1}\int_0^\infty \frac{t^{p-1}\,dt}{1+t}.$$

The integral representation

$$B(u,v) = \int_0^\infty \frac{t^{u-1}\,dt}{(1+t)^{u+v}}$$

shows that the integral is $B(p, 1-p)$. The identity $\Gamma(p)\Gamma(1-p) = \pi/\sin \pi p$ gives the result.