PROOF OF FORMULA 3.223.3

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(b-x)(a-x)} = -\pi\cot\pi\mu \cdot \frac{b^{\mu-1}-a^{\mu-1}}{b-a}$$

The partial fraction decomposition

$$\frac{1}{(b-x)(a-x)} = \frac{1}{a-b}\left(\frac{1}{b-x} - \frac{1}{a-x}\right)$$

gives

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(b-x)(a-x)} = \frac{1}{a-b}\int_0^\infty \frac{x^{\mu-1}\,dx}{b-x} - \frac{1}{a-b}\int_0^\infty \frac{x^{\mu-1}\,dx}{a-x}.$$

Introduce the changes of variables $x = bt$ and $x = at$ in the first and second integral, respectively, to obtain

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(b-x)(a-x)} = \frac{b^{\mu-1}}{a-b}\int_0^\infty \frac{t^{\mu-1}\,dt}{1-t} - \frac{a^{\mu-1}}{a-b}\int_0^\infty \frac{t^{\mu-1}\,dt}{1-t}.$$

This integral has been evaluated in 3.222.2. Its value is

$$\int_0^\infty \frac{t^{\mu-1}\,dt}{1-t} = \frac{\pi\,\cos\pi\mu}{\sin\pi\mu}.$$

This gives the result.