PROOF OF FORMULA 3.224

$$\int_0^\infty \frac{(x+b)\, x^{\mu-1}\, dx}{(x+c)(x+a)} = \frac{\pi}{\sin \pi\mu} \left(\frac{c-b}{c-a} c^{\mu-1} + \frac{a-b}{a-c} a^{\mu-1} \right)$$

The partial fraction decomposition

$$\frac{x+b}{(x+c)(x+a)} = \frac{b-c}{a-c}\frac{1}{x+c} - \frac{b-a}{a-c}\frac{1}{x+a}$$

gives

$$\int_0^\infty \frac{(x+b)\, x^{\mu-1}\, dx}{(x+c)(x+a)} = \frac{b-c}{a-c} \int_0^\infty \frac{x^{\mu-1}\, dx}{x+c} - \frac{b-a}{a-c} \int_0^\infty \frac{x^{\mu-1}\, dx}{x+a}.$$

Let $x = ct$ and $x = at$ in the first and second integral, respectively, and use

$$\int_0^\infty \frac{t^{\mu-1}\, dt}{1+t} = \frac{\pi}{\sin \pi\mu}$$

to obtain the result.