PROOF OF FORMULA 3.231.2

$$\int_0^1 \frac{x^{p-1} + x^{-p}}{1+x}\, dx = \frac{\pi}{\sin \pi p}$$

The incomplete beta function β is defined by

$$\beta(a) = \int_0^1 \frac{x^{a-1}\, dx}{1+x},$$

and it satisfies

$$\beta(a) = \frac{1}{2}\left[\psi\left(\frac{a+1}{2}\right) - \psi\left(\frac{a}{2}\right)\right].$$

Observe that

$$\begin{aligned}
\int_0^1 \frac{x^{p-1} + x^{-p}}{1+x}\, dx &= \beta(p) + \beta(1-p) \\
&= \frac{1}{2}\left[\psi\left(\frac{p}{2} + \frac{1}{2}\right) - \psi\left(\frac{p}{2}\right) + \psi\left(1 - \frac{p}{2}\right) - \psi\left(\frac{1}{2} - \frac{p}{2}\right)\right] \\
&= \frac{\pi}{2}\left(\tan\left(\frac{\pi p}{2}\right) + \cot\left(\frac{\pi p}{2}\right)\right) \\
&= \frac{\pi}{\sin \pi p}
\end{aligned}$$