PROOF OF FORMULA 3.233

$$\int_0^\infty \left(\frac{1}{1+x} - \frac{1}{(1+x)^\nu} \right) \frac{dx}{x} = \psi(\nu) + \gamma$$

The identity

$$\int_0^1 \frac{x^{p-1} - x^{q-1}}{1-x} \, dx = \psi(q) - \psi(p)$$

appears in formula 3.231.5. The change of variables $t = (1-x)/x$ produces the evaluation

$$\int_0^\infty \left(\frac{1}{(1+t)^p} - \frac{1}{(1+t)^q} \right) \frac{dt}{t} = \psi(q) - \psi(p).$$

The special case $p = 1$ and $q = \nu$ yields

$$\int_0^\infty \left(\frac{1}{1+t} - \frac{1}{(1+t)^\nu} \right) \frac{dt}{t} = \psi(\nu) + \gamma.$$