PROOF OF FORMULA 3.238.1

$$\int_{-\infty}^{\infty} \frac{|x|^{\nu-1}\,dx}{x-u} = -\pi \cot\left(\frac{\pi\nu}{2}\right) |u|^{\nu-1} \text{sign } u$$

Let $x = tu$ to obtain

$$\int_{-\infty}^{\infty} \frac{|x|^{\nu-1}\,dx}{x-u} = -|u|^{\nu-1} \text{sign } u \int_{-\infty}^{\infty} \frac{|t|^{\nu-1}}{1-t}\,dt.$$

This last integral is

$$\int_{-\infty}^{\infty} \frac{|t|^{\nu-1}}{1-t}\,dt = \int_0^{\infty} \frac{t^{\nu-1}\,dt}{1+t} + \int_0^{\infty} \frac{t^{\nu-1}\,dt}{1-t}.$$

These integrals are evaluated in 3.241.3 and 3.241.2 respectively. It follows that

$$\int_{-\infty}^{\infty} \frac{|t|^{\nu-1}}{1-t}\,dt = \frac{\pi}{\sin \pi\nu} + \frac{\pi}{\tan \pi\nu}.$$

This simplifies to $\cot(\pi\nu/2)$.