

PROOF OF FORMULA 3.241.2

$$\int_0^\infty \frac{x^{\mu-1} dx}{1+x^\nu} = \frac{1}{\nu} B\left(\frac{\mu}{\nu}, 1 - \frac{\mu}{\nu}\right) = \frac{\pi}{\nu \sin(\pi\mu/\nu)}$$

Let $t = x^\nu$ to obtain

$$\int_0^\infty \frac{x^{\mu-1} dx}{1+x^\nu} = \frac{1}{\nu} \int_0^\infty \frac{t^{\mu/\nu-1} dt}{1+t}.$$

The integral representation

$$B(a, b) = \int_0^\infty \frac{t^{a-1} dt}{(1+t)^{a+b}},$$

gives the value of the integral. The formula is simplified using

$$\Gamma(a)\Gamma(1-a) = \frac{\pi}{\sin \pi a}.$$