PROOF OF FORMULA 3.249.1

$$\int_0^\infty \frac{dx}{(x^2+a^2)^n} = \frac{(2n-3)!!}{(2n-2)!!}\frac{\pi}{2}\frac{1}{a^{2n-1}}$$

Let $t = x/a$ to obtain

$$\int_0^\infty \frac{dx}{(x^2+a^2)^n} = \frac{1}{a^{2n-1}}\int_0^\infty \frac{dt}{(t^2+1)^n}.$$

Wallis' formula

$$\int_0^\infty \frac{dt}{(t^2+1)^n} = \frac{\pi}{2^{2n-1}}\frac{(2n-1)!}{(n-1)!^2}$$

gives the result after using the relation

$$(2m)! = (2m-1)!!2^m m!.$$

Note. This formula generalizes to

$$\int_0^\infty \frac{dx}{(x^2+a^2)^b} = \frac{\sqrt{\pi}\Gamma(b-\frac{1}{2})}{2\Gamma(b)a^{2b-1}}.$$