PROOF OF FORMULA 3.249.2

$$\int_0^a (a^2 - x^2)^{n-1/2}\, dx = a^{2n} \frac{(2n-1)!!}{(2n)!!} \frac{\pi}{2}$$

Let $x = at$ to obtain

$$\int_0^a (a^2 - x^2)^{n-1/2}\, dx = a^{2n} \int_0^1 (1 - t^2)^{n-1/2}\, dt.$$

The change of variables $t = \sin\theta$ gives

$$\int_0^a (a^2 - x^2)^{n-1/2}\, dx = a^{2n} \int_0^{\pi/2} \cos^{2n}\theta\, d\theta,$$

and the result now follows from Wallis' formula

$$\int_0^{\pi/2} \cos^{2n}\theta\, d\theta = \frac{\pi}{2^{2n+1}} \binom{2n}{n} = \frac{\pi}{2} \frac{(2n-1)!!}{(2n)!!}$$