PROOF OF FORMULA 3.251.6

$$\int_0^\infty \frac{x^{\mu+1}\,dx}{(1+x^2)^2} = \frac{\pi\mu}{4\sin(\pi\mu/2)}$$

Let $t = x^2$ to obtain

$$\int_0^\infty \frac{x^{\mu+1}\,dx}{(1+x^2)^2} = \frac{1}{2}\int_0^\infty \frac{t^{\mu/2}\,dt}{(1+t)^2}.$$

The integral representation

$$B(a,b) = \int_0^\infty \frac{t^{a-1}\,dt}{(1+t)^{a+b}},$$

shows that the requested integral is

$$\begin{aligned} \frac{1}{2}B\left(\tfrac{\mu}{2}+1, 1-\tfrac{\mu}{2}\right) &= \frac{\Gamma(1+\mu/2)\,\Gamma(1-\mu/2)}{2\Gamma(2)}, \\ &= \frac{\mu}{2}\Gamma(\mu/2)\Gamma(1-\mu/2) \end{aligned}$$

and the stated result follows from $\Gamma(x)\Gamma(1-x) = \pi/\sin(\pi x)$.