PROOF OF FORMULA 3.251.8

$$\int_0^1 x^{q+p-1}(1-x^q)^{-p/q}\,dx = \frac{\pi p}{q^2\,\sin(\pi p/q)}$$

Let $t = x^q$ to obtain

$$\begin{aligned}\int_0^1 x^{q+p-1}(1-x^q)^{-p/q}\,dx &= \frac{1}{q}\int_0^1 t^{p/q}(1-t)^{-p/q}\,dt \\ &= \frac{1}{q}B\left(1+\frac{p}{q}, 1-\frac{p}{q}\right),\end{aligned}$$

using the integral representation for the beta function

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\,dt.$$

The result is simplified using the identity

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

and

$$\Gamma(a)\Gamma(1-a) = \frac{\pi}{\sin \pi a}.$$