PROOF OF FORMULA 3.251.9

$$\int_0^1 x^{q/p-1}(1-x^q)^{-1/p}\,dx = \frac{\pi}{q\,\sin(\pi/p)}$$

Let $t = x^q$ to obtain

$$\begin{aligned}\int_0^1 x^{q/p-1}(1-x^q)^{-1/p}\,dx &= \frac{1}{q}\int_0^1 t^{1/p-1}(1-t)^{-1/p}\,dt \\ &= \frac{1}{q}B\left(\frac{1}{p}, 1-\frac{1}{p}\right),\end{aligned}$$

using the integral representation for the beta function

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\,dt.$$

The result is simplified using the identity

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

and

$$\Gamma(a)\Gamma(1-a) = \frac{\pi}{\sin \pi a}.$$