PROOF OF FORMULA 3.252.2

$$\int_{-\infty}^{\infty} \frac{dx}{(ax^2+2bx+c)^n} = \pi \, \frac{(2n-3)!!}{(2n-2)!!} \, \frac{a^{n-1}}{(ac-b^2)^{n-1/2}}$$

Complete the square in the form

$$ax^2+2bx+c = a\left[\left(x + \frac{b}{a} \right)^2 + \frac{ac-b^2}{a^2} \right],$$

let

$$u = \frac{a}{\sqrt{ac-b^2}} \left(x + \frac{b}{a} \right),$$

and use the symmetry of the integrand to get

$$\int_{-\infty}^{\infty} \frac{dx}{(ax^2+2bx+c)^n} = \frac{2a^{n-1}}{(ac-b^2)^{n-1/2}} \int_0^{\infty} \frac{du}{(u^2+1)^n}.$$

This integral is evaluated in 3.249.1, it corresponds to Wallis' formula. Its value is

$$\int_0^{\infty} \frac{du}{(u^2+1)^n} = \frac{(2n-3)!!}{(2n-2)!!} \times \frac{\pi}{2}.$$