PROOF OF FORMULA 3.267.2

$$\int_0^1 \frac{x^{3n-1}\,dx}{\sqrt[3]{1-x^3}} = \frac{\Gamma(n)\Gamma\left(\frac{2}{3}\right)}{3\Gamma\left(n+\frac{2}{3}\right)} = \frac{(n-1)!}{3\left(\frac{2}{3}\right)_n}$$

Let $t = x^3$ to obtain

$$\int_0^1 \frac{x^{3n-1}\,dx}{\sqrt[3]{1-x^3}} = \frac{1}{3}\int_0^1 \frac{t^{n-1}\,dt}{(1-t)^{1/3}}.$$

The integral representation

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\,dt$$

gives the last integral as

$$B\left(n,\tfrac{2}{3}\right) = \frac{\Gamma(n)\Gamma(\frac{2}{3})}{\Gamma(n+\frac{2}{3})}.$$

The result is simplified using

$$\Gamma(a)\Gamma(1-a) = \frac{\pi}{\sin \pi a}.$$