**PROOF OF FORMULA 3.272.2**

$$\int_0^1 \frac{x^{n-1}+x^{n-2/3}+x^{n-1/3}-3x^{3n-1}}{1-x}\,dx = 3\ln 3$$

Write the integral as

$$\int_0^1 \frac{x^{n-1}+x^{n-2/3}+x^{n-1/3}-3x^{3n-1}}{1-x}\,dx = \int_0^1 \frac{x^{n-1}-x^{3n-1}}{1-x}\,dx + \int_0^1 \frac{x^{n-2/3}-x^{3n-1}}{1-x}\,dx + \int_0^1 \frac{x^{n-1/3}-x^{3n-1}}{1-x}\,dx.$$

Using entry 3.231.5

$$\int_0^1 \frac{x^{\mu-1}-x^{\nu-1}}{1-x}\,dx = \psi(\nu)-\psi(\mu)$$

it follows that

$$\int_0^1 \frac{x^{n-1}+x^{n-2/3}+x^{n-1/3}-3x^{3n-1}}{1-x}\,dx = 3\psi(3n)-\left(\psi(n)+\psi(n-1/3)+\psi(n-2/3)\right).$$

The identity

$$\psi(Nt) = \frac{1}{N}\sum_{k=0}^{N-1}\psi(t+k/N)+\ln N$$

gives the result.