PROOF OF FORMULA 3.311.6

$$\int_0^\infty \frac{e^{-\mu x} - e^{-x}}{1 - e^{-x}}\, dx = \psi(\mu) + \gamma$$

Let $t = e^{-x}$ to obtain

$$\int_0^\infty \frac{e^{-\mu x} - e^{-\nu x}}{1 - e^{-x}}\, dx = \int_0^1 \frac{t^{\mu-1} - t^{\nu-1}}{1-t}\, dt.$$

The result now follows from the representation

$$\psi(a) = -\int_0^1 \left(\frac{1}{\ln t} + \frac{t^{a-1}}{1-t} \right) dt,$$

and the special value for $\nu = 1:\ \psi(1) = -\gamma$.