PROOF OF FORMULA 3.315.2

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\, dx}{(a+e^{-x})(b+e^{-x})} = -\frac{(a^{\mu-1}-b^{\mu-1})}{a-b}\,\frac{\pi}{\sin \pi\mu}$$

The change of variables $t = e^{-x}$ gives

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\, dx}{(a+e^{-x})(b+e^{-x})} = \int_0^{\infty} \frac{t^{\mu-1}\, dt}{(a+t)(b+t)}.$$

Partial fraction decomposition of the integrand produces

$$\int_0^{\infty} \frac{t^{\mu-1}\, dt}{(a+t)(b+t)} = \frac{1}{b-a}\int_0^{\infty} \frac{t^{\mu-1}\, dt}{t+a} - \frac{1}{b-a}\int_0^{\infty} \frac{t^{\mu-1}\, dt}{t+b}.$$

Let $t = as$ in the first integral and $t = bs$ in the second one. The value of

$$\int_0^{\infty} \frac{s^{\mu-1}\, ds}{1+s} = B(\mu, 1-\mu) = \Gamma(\mu)\Gamma(1-\mu) = \frac{\pi}{\sin \pi\mu},$$

gives the result.