PROOF OF FORMULA 3.334

$$\int_0^\infty (e^x-1)^{\nu-1}\exp\left[-\frac{b}{e^x-1}-\mu x\right]\,dx = \Gamma(\mu-\nu-1)e^{b/2}b^{\frac{\nu-1}{2}}W_{\frac{\nu-2\mu-1}{2},-\frac{\nu}{2}}(b)$$

The change of variables $t = 1/(e^x-1)$ gives

$$\int_0^\infty (e^x-1)^{\nu-1}\exp\left[-\frac{b}{e^x-1}-\mu x\right]\,dx = \int_0^\infty t^{\mu-\nu}(1+t)^{-1-\mu}e^{-bt}\,dt.$$

The Whittaker function is defined by the integral representation

$$W_{a,b}(z) = \frac{z^{b+1/2}e^{-z/2}}{\Gamma(b-a+\frac{1}{2})}\int_0^\infty e^{-zt}\,t^{b-a-\frac{1}{2}}\,(1+t)^{b+a-\frac{1}{2}}.$$

Matching parameters yields the evaluation.