PROOF OF FORMULA 3.411.13

$$\int_0^\infty \frac{x\, e^{-(2n-1)x}\, dx}{1+e^x} = -\frac{\pi^2}{12} + \sum_{j=1}^{2n-1} \frac{(-1)^{j-1}}{j^2}$$

Formula 3.411.8 states that

$$\int_0^\infty \frac{x^{n-1} e^{-px}\, dx}{1+e^x} = (n-1)! \sum_{k=1}^\infty \frac{(-1)^{k-1}}{(p+k)^n}.$$

Put $n=2$ and then $p = 2n-1$ to obtain

$$\begin{aligned} \int_0^\infty \frac{xe^{-(2n-1)x}\, dx}{1+e^x} &= \sum_{k=1}^\infty \frac{(-1)^{k-1}}{(k+2n-1)^2} \\ &= \sum_{j=2n}^\infty \frac{(-1)^j}{j^2} \\ &= \sum_{j=1}^\infty \frac{(-1)^j}{j^2} - \sum_{j=1}^{2n-1} \frac{(-1)^j}{j^2} \end{aligned}$$

and the result follows from

$$\sum_{j=1}^\infty \frac{(-1)^j}{j^2} = -\frac{\pi^2}{12}.$$