PROOF OF FORMULA 3.411.2

$$\int_0^\infty \frac{x^{2n-1}\,dx}{e^{ax}-1} = (-1)^{n-1}\left(\frac{2\pi}{a}\right)^{2n}\frac{B_{2n}}{4n}$$

From formula 3.411.1 it follows that

$$\int_0^\infty \frac{x^{2n-1}\,dx}{e^{ax}-1} = \frac{\Gamma(2n)\zeta(2n)}{a^{2n}}.$$

The result now follows from the representation of the values of the Riemann zeta function at even integers in terms of the Bernoulli numbers:

$$\zeta(2n) = \frac{(-1)^{n-1}B_{2n}(2\pi)^{2n}}{2(2n)!}.$$