PROOF OF FORMULA 3.412

$$\int_0^\infty \left\{ \frac{a + b e^{-px}}{c e^{px} + g + h e^{-px}} - \frac{a + b e^{-qx}}{c e^{qx} + g + h e^{-qx}} \right\} \, \frac{dx}{x} = \frac{a + b}{c + g + h} \, \ln \frac{p}{q}$$

Frullani formula states that

$$\int_0^\infty \frac{f(px) - f(qx)}{x} dx = [f(0) - f(\infty)] \ln \left(\frac{q}{p}\right).$$

This formula corresponds to choosing

$$f(x) = \frac{a + be^{-x}}{ce^x + g + he^{-x}}.$$