FORMULA 3.417.1

$$\int_{-\infty}^{\infty} \frac{x\,dx}{a^2e^x + b^2e^{-x}} = \frac{\pi}{2ab} \ln\left(\frac{b}{a}\right)$$

Let $u = ae^x/b$ to obtain

$$\int_{-\infty}^{\infty} \frac{x\,dx}{a^2e^x + b^2e^{-x}} = \frac{1}{ab} \int_0^{\infty} \frac{\ln(au/b)\,du}{u^2+1}$$

that can be separated as

$$\int_{-\infty}^{\infty} \frac{x\,dx}{a^2e^x + b^2e^{-x}} = \frac{1}{ab} \left(\int_0^{\infty} \frac{\ln u\,du}{u^2+1} + \ln\left(\frac{b}{a}\right) \int_0^{\infty} \frac{du}{u^2+1} \right).$$

The first integral vanishes: this follows directly from the change of variables $x \mapsto 1/x$. The second integral is $\pi/2$.