

PROOF OF FORMULA 3.418.3

$$\int_0^{\ln 2} \frac{x dx}{e^x + 2e^{-x} - 2} = \frac{\pi}{8} \ln 2$$

The change of variables $u = e^x$ gives

$$\int_0^{\ln 2} \frac{x dx}{e^x + 2e^{-x} - 2} = \int_1^2 \frac{\ln u du}{u^2 - 2u + 2}.$$

Now let $u = z + 1$ produces

$$\int_0^{\ln 2} \frac{x dx}{e^x + 2e^{-x} - 2} = \int_0^1 \frac{\ln(z + 1) dz}{z^2 + 1}.$$

The value of this integral is the special case $a = 1$ of 4.291.18

$$\int_0^a \frac{\ln(1 + az) dz}{1 + z^2} = \frac{1}{2} \tan^{-1} a \ln(1 + a^2).$$