PROOF OF FORMULA 3.471.16

$$\int_0^\infty x^{n-1/2} e^{-px-q/x}\, dx = (-1)^n \sqrt{\pi} \frac{\partial^n}{\partial p^n} \left[p^{-1/2} e^{-2\sqrt{pq}} \right]$$

Define

$$I_n(p) := \int_0^\infty x^{n-1/2} e^{-px-q/x}\, dx.$$

Formula 3.471.15 gives the value of $I_0(p)$ and the relation

$$\frac{\partial I_n(p)}{\partial p} = -I_{n+1}(p)$$

gives the result for $n > 0$.