PROOF OF FORMULA 3.512.1

$$\int_0^\infty \frac{\cosh 2bx}{(\cosh ax)^{2\nu}}\, dx = \frac{4^{\nu-1}}{a} B\left(\nu + \frac{b}{a}, \nu - \frac{b}{a}\right)$$

Let $t = ax$ and replace b/a by c to get an equivalent form of the entry:

$$\int_0^\infty \frac{\cosh 2ct}{(\cosh t)^{2\nu}}\, dt = 4^{\nu-1} B(\nu + c, \nu - c).$$

To prove this, write the integrand in exponential form and let $u = e^{-2t}$ to obtain

$$\int_0^\infty \frac{e^{2(c-\nu)t} + e^{-2(c+\nu)t}}{(1+e^{-2t})^{2\nu}}\, dt = \int_0^1 \frac{u^{\nu+c} + u^{\nu-c}}{(1+u)^{2\nu}}\, du.$$

The result now comes from the integral representation (entry 8.380.5):

$$B(x,y) = \int_0^1 \frac{u^{x-1} + u^{y-1}}{(1+u)^{x+y}}\, du.$$