PROOF OF FORMULA 3.515

$$\int_{-\infty}^{\infty} \left(1 - \frac{\sqrt{2} \cosh x}{\sqrt{\cosh 2x}} \right) dx = -\ln 2$$

Let $t = e^{2x}$ to obtain

$$\int_{-\infty}^{\infty} \left(1 - \frac{\sqrt{2} \cosh x}{\sqrt{\cosh 2x}} \right) dx = \int_{1}^{\infty} \left(1 - \frac{t+1}{\sqrt{t^2+1}} \right) \frac{dt}{t}.$$

The change of variable $t = \tan \varphi$ gives

$$\int_{-\infty}^{\infty} \left(1 - \frac{\sqrt{2} \cosh x}{\sqrt{\cosh 2x}} \right) dx = \int_{\pi/4}^{\pi/2} \frac{1 - \cos \varphi - \sin \varphi}{\cos \varphi \, \sin \varphi} \, d\varphi.$$

Now let $s = \tan \varphi/2$ to produce

$$\int_{\pi/4}^{\pi/2} \frac{1 - \cos \varphi - \sin \varphi}{\cos \varphi \, \sin \varphi} \, d\varphi = -2 \int_{\tan \pi/8}^{1} \frac{ds}{s+1}.$$

The result follows from the value $\tan \frac{\pi}{8} = \sqrt{2} - 1$.