The original formula is

$$
\int_{0}^{\infty}\left(\cos \left(a x^{2}\right)-\sin \left(a x^{2}\right)\right) \frac{d x}{x^{4}+b^{4}}=\frac{\pi e^{-a b^{2}}}{2 b^{3} \sqrt{2}}
$$

the change of variables $x=b t$ and replacing $a b^{2}$ by $a$ (and going back to $x$ as the integration variable) gives the new formula

$$
\int_{0}^{\infty}\left(\cos \left(a x^{2}\right)-\sin \left(a x^{2}\right)\right) \frac{d x}{x^{4}+1}=\frac{\pi e^{-a}}{2 \sqrt{2}}
$$

