PROOF OF FORMULA 4.252.4

$$\int_0^\infty \frac{x^{\mu-1}\,\ln x}{(x+a)^2}\,dx = \frac{(1-\mu)a^{\mu-2}\pi}{\sin\pi\mu}\left(\ln a - \cot\pi\mu + \frac{1}{\mu-1}\right)$$

Entry 4.251.1 states that

$$\int_0^\infty \frac{x^{\mu-1}\,\ln x}{a+x}\,dx = \frac{\pi a^{\mu-1}}{\sin\pi\mu}(\ln a - \pi\cot\pi\mu).$$

The result is obtained by differentiating with respect to a.