PROOF OF FORMULA 4.254.1

$$\int_0^1 \frac{x^{p-1}\,\ln x}{1-x^q}\,dx = -\frac{1}{q^2}\psi'\left(\frac{p}{q}\right)$$

The change of variable $t = x^q$ gives

$$\int_0^1 \frac{x^{p-1}\,\ln x}{1-x^q}\,dx = \frac{1}{q^2}\int_0^1 \frac{t^{p/q-1}\,\ln t}{1-t}dt.$$

The result now follows from the formula

$$\int_0^1 \frac{x^{a-1}\,\ln x}{1-x}\,dx = -\psi'(a)$$

established in the proof of entry 4.252.3.