# PROOF OF FORMULA 4.267.15

$$\int_0^1 \frac{x^p - x^q}{1-x} \, \frac{1-x^r}{\ln x} \, dx = \ln \left[ \frac{\Gamma(q+1)\,\Gamma(p+r+1)}{\Gamma(p+1)\,\Gamma(q+r+1)} \right]$$

Entry 3.231.5 states that

$$\int_0^1 \frac{x^{a-1} - x^{b-1}}{1-x} \, dx = \psi(b) - \psi(a).$$

Integrate with respect to $a$ from $a_1$ to $a_2$ to produce

$$Y(a_2, a_1) := \int_0^1 \frac{x^{a_2-1} - x^{a_1-1}}{(1-x)\,\ln x} \, dx = \ln \left[ \frac{\Gamma(a_1)}{\Gamma(a_2)} \right].$$

Therefore,

$$\int_0^1 \frac{x^p - x^q}{1-x} \, \frac{1-x^r}{\ln x} \, dx = Y(p,q) - Y(p+r, q+r).$$

The result now follows by replacing the explicit form of $Y$.