PROOF OF FORMULA 4.267.23

$$\int_0^\infty \frac{x^{p-1}-x^{q-1}}{1-x^{2n}} \frac{1-x^2}{\ln x} \, dx = \ln \left[\frac{\sin(\frac{\pi p}{2n}) \sin(\frac{\pi(q+2)}{2n})}{\sin(\frac{\pi q}{2n}) \sin(\frac{\pi(p+2)}{2n})} \right]$$

Entry 4.267.19 states that

$$\int_0^\infty \frac{x^{p-1}-x^{q-1}}{(1-x^r)\,\ln x} \, dx = \ln \left[\frac{\sin(\frac{\pi p}{r})}{\sin(\frac{\pi q}{r})} \right].$$

Write the current integral as

$$\int_0^\infty \frac{x^{p-1}-x^{q-1}}{1-x^{2n}} \frac{1-x^2}{\ln x} \, dx = \int_0^\infty \frac{x^{p-1}-x^{q-1}}{1-x^{2n}} \frac{dx}{\ln x} - \int_0^\infty \frac{x^{p+1}-x^{q+1}}{1-x^{2n}} \frac{dx}{\ln x},$$

and use the formula above to obtain the result.