PROOF OF FORMULA 4.272.9

$$\int_0^1 \left(\ln \frac{1}{x}\right)^{n-1} \frac{x^{\nu-1}}{1-x}\, dx = (n-1)!\, \zeta(n,\nu)$$

Expand the integrand in a geometric series and use the change of variables $u = \ln 1/x$ to produce

$$\begin{aligned} \int_0^1 \left(\ln \frac{1}{x}\right)^{n-1} \frac{x^{\nu-1}}{1-x}\, dx &= \sum_{k=0}^{\infty} \left(\ln \frac{1}{x}\right)^{n-1} x^{\nu-1+k}\, dx \\ &= \sum_{k=0}^{\infty} \int_0^{\infty} u^{n-1} e^{-(\nu+k)u}\, du. \end{aligned}$$

The change of variables $t = (\nu+k)u$ gives the result.