PROOF OF FORMULA 4.281.3

$$\int_0^1 \frac{x^{p-1}\,dx}{q \pm \ln x} = \pm e^{\mp pq}\,\text{Ei}\,(\pm pq)$$

We compute the case of + sign, the other one is similar. The change of variables $u = q + \ln x$ yields

$$\int_0^1 \frac{x^{p-1}\,dx}{q \pm \ln x} = e^{-pq} \int_{-\infty}^{q} \frac{e^{up}}{u}\,du.$$

The result now follows from the change of variables $t = up$ and the definition of the exponential integral

$$\text{Ei}(x) = \int_{-\infty}^{x} \frac{e^t}{t}\,dt.$$