PROOF OF FORMULA 4.291.12

$$\int_0^1 \frac{\ln(1+x)}{x(1+x)}\,dx = \frac{\pi^2}{12} - \frac{1}{2}\ln^2 2$$

Expand in partial fractions to obtain

$$\int_0^1 \frac{\ln(1+x)}{x(1+x)}\,dx = \int_0^1 \frac{\ln(1+x)}{x}\,dx - \int_0^1 \frac{\ln(1+x)}{1+x}\,dx.$$

Entry 4.291.1 gives

$$\int_0^1 \frac{\ln(1+x)}{x}\,dx = \frac{\pi^2}{12}$$

and entry 4.291.6 gives

$$\int_0^1 \frac{\ln(1+x)}{1+x}\,dx = \frac{1}{2}\ln^2 2.$$

The result follows from there.