**FORMULA 4.291.14**

$$\int_0^1 \frac{\ln(1+x)}{(ax+b)^2}\,dx = \frac{1}{a(a-b)}\ln\left(\frac{a+b}{b}\right) + \frac{2\ln 2}{b^2-a^2} \qquad \text{if } a \neq b$$

$$\int_0^1 \frac{\ln(1+x)}{(ax+b)^2}\,dx = \frac{1-\ln 2}{2a^2} \qquad \text{if } a = b$$

Integrate by parts to produce

$$\int_0^1 \frac{\ln(1+x)}{(ax+b)^2}\,dx = -\frac{\ln 2}{a(a+b)} + \frac{1}{a}\int_0^1 \frac{dx}{(1+x)(ax+b)}.$$

The result now follows from the partial fraction decomposition

$$\frac{1}{(1+x)(ax+b)} = \frac{1}{b-a}\frac{1}{1+x} - \frac{b}{b-a}\frac{1}{ax+b},$$

valid for $a \neq b$.

In the case $a = b$ we have

$$\int_0^1 \frac{\ln(1+x)}{(ax+b)^2}\,dx = -\frac{\ln 2}{a^2} + \frac{1}{a^2}\int_0^1 \frac{dx}{(1+x)^2},$$

that yields the stated result.