PROOF OF FORMULA 4.291.16

$$\int_0^1 \frac{\ln(a+x)}{a+x^2}\,dx = \frac{1}{2\sqrt{a}}\cot^{-1} a\,\ln(a(1+a))$$

The change of variable $x = \sqrt{a}t$ gives

$$\int_0^1 \frac{\ln(a+x)}{a+x^2}\,dx = \frac{\ln a}{\sqrt{a}}\int_0^{1/\sqrt{a}} \frac{dt}{1+t^2} + \frac{1}{\sqrt{a}}\int_0^{1/\sqrt{a}} \frac{\ln(1+t/\sqrt{a})}{1+t^2}\,dt.$$

The first integral is elementary and the second one is entry 4.291.18

$$\int_0^b \frac{\ln(1+bx)\,dx}{1+x^2} = \frac{1}{2}\tan^{-1} b\,\ln(1+b^2)$$

with $b = 1/\sqrt{a}$. This gives the result