PROOF OF FORMULA 4.291.20

$$\int_0^1 \frac{\ln(ax+b)}{(1+x)^2}\,dx = \frac{1}{2(a-b)}\left[(a+b)\ln(a+b) - 2b\ln b - 2a\ln 2\right]$$

Integrate by parts to get

$$\int_0^1 \frac{\ln(ax+b)}{(1+x)^2}\,dx = -\frac{1}{2}\ln(a+b) + \ln b + a\left(\frac{1}{b-a}\int_0^1 \frac{dx}{1+x} - \frac{a}{b-a}\int_0^1 \frac{dx}{ax+b}\right).$$

The result follows by evaluating the elementary integrals.