

PROOF OF FORMULA 4.291.9

$$\int_0^{\infty} \frac{\ln(1+x)}{1+x^2} dx = \frac{\pi}{4} \ln 2 + G$$

Split the integral at $t = 1$ and in the part $t \geq 1$ change t by $1/t$ to obtain

$$\int_0^{\infty} \frac{\ln(1+x)}{1+x^2} dx = 2 \int_0^1 \frac{\ln(1+t)}{1+t^2} dx - \int_0^1 \frac{\ln t}{1+t^2} dt.$$

The first integral is in entry 4.291.8 with value $\frac{1}{8}\pi \ln 2$. The second one appears as the case $a = 1$ in entry 4.231.11 with value $-G$.