PROOF OF FORMULA 4.293.11

$$\int_0^\infty \frac{x^{\mu-1}\ \ln(1+x)}{1+x}\,dx = -\frac{\pi}{\sin\pi\mu}\,(\gamma+\psi(1-\mu))$$

Entry 4.293.14 states that

$$\int_0^\infty \frac{x^{\mu-1}\ln(a+x)}{(x+a)^\nu}\,dx = a^{\mu-\nu}B(\mu,\nu-\mu)\left[\psi(\nu)-\psi(\nu-\mu)+\ln a\right].$$

The special case $a=\nu=1$ gives

$$\int_0^\infty \frac{x^{\mu-1}\ln(1+x)}{x+1}\,dx = B(\mu,1-\mu)\left[\psi(1)-\psi(1-\mu)\right].$$

The result now follows from

$$B(\mu,1-\mu)=\Gamma(\mu)\Gamma(1-\mu)=\frac{\pi}{\sin\pi\mu}$$

and $\psi(1)=-\gamma$.