PROOF OF FORMULA 4.293.13

$$\int_0^1 \frac{x^{\mu-1} \ln(1-x)}{(1-x)^{1-\nu}} dx = B(\mu,\nu) \left[\psi(\nu) - \psi(\mu+\nu) \right]$$

Start with

$$B(\mu, \nu) = \int_0^1 x^{\mu - 1} (1 - x)^{\nu - 1} dx$$

and differentiate with respect to ν to produce

$$\frac{d}{d\nu}B(\mu,\nu) = B(\mu,\nu)\left[\psi(\nu) - \psi(\mu+\nu)\right].$$

This is the result.