PROOF OF FORMULA 4.293.5

$$\int_0^1 x^{2n} \ln(x+1)\, dx = \frac{1}{2n+1}\left[2\ln 2 + \sum_{k=1}^{2n+1} \frac{(-1)^k}{k} \right]$$

Integrate by parts to obtain

$$\int_0^1 x^{2n} \ln(x+1)\, dx = \frac{\ln 2}{2n+1} - \frac{1}{2n+1}\int_0^1 \frac{x^{2n+1}\, dx}{1+x}.$$

Now

$$\int_0^1 \frac{x^{2n+1}\, dx}{1+x} = \int_0^1 \frac{x^{2n+1}+1}{1+x}\, dx - \int_0^1 \frac{dx}{1+x}\, dx.$$

The first integral is

$$\int_0^1 \frac{x^{2n+1}+1}{1+x}\, dx = \sum_{k=0}^{2n} (-1)^k \int_0^1 x^k\, dx = \sum_{k=0}^{2n} \frac{(-1)^k}{k+1}.$$

The second one is $\ln 2$. The result follows from here.