PROOF OF FORMULA 4.295.13

$$\int_0^1 \frac{\ln(1-x^2)}{1+x^2}\,dx = \frac{\pi}{4}\ln 2 - G$$

The change of variables $x = \tan\varphi$ gives

$$\int_0^1 \frac{\ln(1-x^2)}{1+x^2}\,dx = \int_0^{\pi/4} \ln(\cos\varphi+\sin\varphi)\,d\varphi + \int_0^{\pi/4} \ln(\cos\varphi-\sin\varphi)\,d\varphi - 2\int_0^{\pi/4} \ln\cos\varphi\,d\varphi.$$

The result now follows from the values

$$\int_0^{\pi/4} \ln(\cos\varphi+\sin\varphi)\,d\varphi = -\frac{\pi}{8}\ln 2 + \frac{G}{2}$$

and

$$\int_0^{\pi/4} \ln(\cos\varphi-\sin\varphi)\,d\varphi = -\frac{\pi}{8}\ln 2 - \frac{G}{2}$$

and

$$\int_0^{\pi/4} \ln\cos\varphi\,d\varphi = -\frac{\pi}{4}\ln 2 + \frac{G}{2}$$

given as entries 4.225.4, 4.225.1 and 4.224.5 respectively.