PROOF OF FORMULA 4.297.3

$$\int_0^1 \ln\left(\frac{1-x}{x}\right) \frac{dx}{1+x^2} = \frac{\pi}{8}\ln 2$$

The change of variable $x = \tan\varphi$ gives

$$\int_0^1 \ln\left(\frac{1-x}{x}\right) \frac{dx}{1+x^2} = \int_0^{\pi/4} \ln\left(\cos\varphi - \sin\varphi\right)\, d\varphi - \int_0^{\pi/4} \ln\sin\varphi \, d\varphi.$$

These integrals are

$$\int_0^{\pi/4} \ln\left(\cos\varphi - \sin\varphi\right)\, d\varphi = -\frac{\pi}{8}\ln 2 - \frac{G}{2}$$

and

$$\int_0^{\pi/4} \ln\sin\varphi \, d\varphi = -\frac{\pi}{4}\ln 2 - \frac{G}{2}$$

given as entries 4.225.1 and 4.224.2, respectively. This gives the result.