PROOF OF FORMULA 4.297.7

$$\int_0^\infty \frac{b\ln(1+ax) - a\ln(1+bx)}{x^2}\, dx = ab\ln\frac{b}{a}$$

With $f(t) = \ln(1+t)/t$, the integral is
$$\int_0^\infty \frac{b\ln(1+ax) - a\ln(1+bx)}{x^2}\, dx = ab\int_0^\infty \frac{f(ax) - f(bx)}{x}\, dx.$$
This is an integral of Frullani type with value
$$\int_0^\infty \frac{f(ax) - f(bx)}{x}\, dx = [f(0) - f(\infty)]\ln\frac{b}{a}.$$
The result follows from $f(0) = 1$ and $f(\infty) = 0$.